*Reply by R. L. Kyhl*

I have no disagreement with the comments of L. J. Kaplan and D. J. R. Stock. I was thinking of plotting the two parts of the chart from different origins. My chief interest was in the type of graphical display chosen.

While the issue was at the press, a similar use of a double chart was published elsewhere.[10]

[10] R. M. Steere, "Novel applications of the Smith chart," *Microwave J.*, vol. 3, pp. 97–100; March, 1960.

## Scattering Matrix for an N-Port Power-Divider Junction*

### Introduction

During the course of an investigation of a data-processing technique yielding effectively reduced sidelobes and beamwidth for a microwave radar antenna, the need arose for multiport power dividers. In order to avoid an undesirable decrease in the signal-to-noise ratio, it was necessary that these dividers waste no power. Consequently, a scattering matrix was sought which would have the obvious requirement that there be no wave reflected in the input port and which would allow the power to be divided into arbitrary but fixed relative parts.

It should be noted that there now exist two methods[1] for synthesizing an $n$-port junction at a single frequency directly from the normalized scattering matrix, without use of the associated impedance matrix.

### The Scattering Matrix

A reciprocal, lossless junction can be represented by a symmetric, unitary scattering matrix $\mathbf{S}'$. It is sufficient to consider a real matrix first without losing generality, because the matrix $\mathbf{S}'$ for the general case (including phase shifts) can be derived from the real case by a simple transformation.[2]

The purpose of this paper is to find the scattering matrix for an $n$-port junction, such that when a wave is fed into, say, port one, there will be no reflected wave in that port, and such that the amplitude of the wave transmitted to port $k$ is equal to a given $x_k$. Analytically expressed, this requires that

$$x_k = \sum_i S_{ki}\delta_{1i} = S_{k1}, \qquad (1)$$

where the $x_k$ are subject to the restriction

$$x_1 = 0 \quad \text{and} \quad \sum_{k=2}^{n} x_k^2 = 1. \qquad (2)$$

* Received by the PGMTT, July 15, 1960.

[1] D. C. Youla, "Direct single frequency synthesis from a prescribed scattering matrix," IRE Trans. on Circuit Theory, vol. CT-6, pp. 340–344; December, 1959.

[2] "Reference Data for Radio Engineers," American Book—Stratford Press, Inc., New York, N. Y., 4th ed.; 1956.

Symmetry requires that

$$S_{ij} = S_{ji}, \qquad (3)$$

and unitarity, which reduces to orthogonality for the real case considered here, requires that

$$\sum_{k=1}^{n} (\tilde{S})_{ik} S_{kj} = \sum_{k=1}^{n} S_{ik}S_{kj} = \delta_{ij} \qquad (4)$$

[where use has been made of (3)].

the matrix $\mathbf{S}$ satisfying (1), (3), and (4) may be obtained as follows:

$$\left.\begin{array}{ll} \text{A. } S_{11} = 0, & \\ \text{B. } S_{kk} = x_k^2 - 1 & \text{for } k > 1, \\ \text{C. } S_{ij} = S_{ji} = x_i x_j & \text{for } i \neq j,\, i > 1,\, j > 1, \\ \text{D. } S_{1i} = S_{i1} = x_i & \text{for } i > 1. \end{array}\right\} \qquad (5)$$

To prove that (5) is the required solution, it is only necessary to verify that it satisfies (1), (3), and (4). From $A$ and $D$, it is evident that (1) is satisfied, and from $C$ and $D$ it is evident that (3) is satisfied. To verify that (4) is also satisfied, it is necessary to consider separately various possible values of $i$ and $j$, because of the special nature of the various $S_{ij}$.

1) $i=j=1$.

Using (2) and the rules given in (5),

$$\sum_{k=1}^{n} S_{1k}S_{k1} = \sum_{k=2}^{n} x_k^2 = 1,$$

which satisfies (4).

2) $i=j\neq 1$.

Proceeding as above,

$$\sum_{k=1}^{n} S_{ik}S_{ki} = S_{i1}^2 + S_{ii}^2 + \sum_{k\neq 1,i} S_{ik}S_{ki}$$
$$= x_i^2 + (x_i^2 - 1)^2 + x_i^2 \sum_{k\neq i} x_k^2.$$

According to (2), the above sum on $k$ is $(1-x_i^2)$ and hence the right side reduces to one as required.

3) $i\neq j$.

$$\sum_{k=1}^{n} S_{ik}S_{kj} = S_{i1}S_{1j} + S_{ii}S_{ij} + S_{ij}S_{jj} + \sum_{k\neq 1,i,j} S_{ik}S_{kj}. \qquad (6)$$

It is necessary to consider separately the case $i=1$ (or $j=1$) and $i\neq 1\neq j$.

a) $i=1$.

For this case, the above expression re-reduces to

$$\sum_{k=1}^{n} S_{1k}S_{kj} = x_j(x_j^2 - 1) + x_j \sum_{k\neq j} x_k^2 = 0.$$

The case $j=1$ is essentially the same as that above and therefore (4) is satisfied in both of these cases.

b) $i\neq 1\neq j$.

For this case, (6) becomes

$$\sum_{k=1}^{n} S_{ik}S_{kj} = x_i x_j + x_i x_j(x_i^2 + x_j^2 - 2) + x_i x_j \sum_{k\neq i,j} x_k^2.$$

When use is made of (2), the above equation reduces to zero as required.

### Conclusion

It has been shown that it is theoretically possible to provide a junction which will divide an input wave into many output waves of arbitrary but fixed relative amplitudes. The arbitrariness of the power division means that one may choose any ratios for the output waves and have no reflected wave in the input port. Once such a divider is constructed, however, the ratio of the output waves is fixed.

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## Lossy Resonant Slot Coupling*

The paper by Allen and Kino[1] suggests a novel method of combating troublesome cutoff oscillations in periodic slow-wave structures. The idea is to couple loss periodically into the system through slots which are resonant at the center of the (narrow) oscillation range. The high $Q$ of the slots will effectively decouple the loss in the operating range of the pass band.

To develop this idea, we start with Allen and Kino's (13) for the voltage $\phi(x)$ along the slot in terms of the tangential $\hat{H}$ field along the slot. With respect to their Fig. 2 coordinates, shown in Fig. 1 below, we can

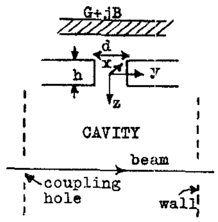


Fig. 1—A lossy slot in a cavity wall. Voltage $\phi(x)$ exists across the slot gap, $d$.

write the transmission line equation for the slot voltage as

$$(\partial^2/\partial x^2 + k^2)\phi(x) = -j\omega L_0[(I_c\hat{H}_{cx})_+ - \hat{H}_{x-}] \qquad (1)$$

where the tangential magnetic field is $(I_c\hat{H}_{cx})_+$ on the $+z$- or cavity side of the slot and is $\hat{H}_{x-}$ on the $-z$-side. Eq. (1) can be derived rigorously for a TEM slot mode. The caret denotes a total field, and we have split the cavity field into an amplitude $I_c$ and a vector field pattern $\hat{H}_c$ for equivalent circuit purposes; this notation differs from that of Allen and Kino. $L_0$ is the slot inductance per unit length in the $x$-direction.

Let us introduce the lossy susceptance $G+jB$ by saying that the average voltage

* Received by the PGMTT, July 20, 1960.

[1] M. A. Allen and G. S. Kino, "On the theory of strongly coupled cavity chains," IRE Trans. on Microwave Theory and Techniques, vol. MTT-8, pp. 362–372; May, 1960.